A FUNDAMENTAL PROPERTY OF MOS TRANSISTORS
(AND ITS CIRCUIT IMPLICATIONS)

E. Vittoz, EPFL, Lausanne
eric.vittoz@ieee.org

EKV 2.6 User's meeting, EPF-Lausanne, November 4, 2004
INTRODUCTION

• Goals of compact transistor modeling:
  • simulation by quantitative calculation on computer
  • **highlighting properties** to facilitate
    - **understanding** circuits
    - synthesis of **robust** circuits

• Best models: combine both goals by hierarchical structure example: EKV model.

• EKV approach will be used to introduce and discuss a **fundamental property**. [1]
DEFINITIONS

for EKV model [2,3]

• Substrate referred-voltages $V_S$, $V_D$, $V_G$

• Position $x$ along the channel

• Local "channel voltage" $V$

splitting of quasi-Fermi levels due non-0 $V_S$ and/or $V_D$

$V = V_S$ at source

$V = V_D$ at drain

n-channel: holes at equilibrium

thus $V = $ electron quasi-Fermi level + constant.
• For a long and wide channel:

\[ I_D = \mu W(-Q_i) \frac{dV}{dx} = \frac{F(V, V_G)dV}{G(x, V_G)dx} \]

• Condition: separable in \( V \) and \( x \)

\[ \int_0^L \mathbf{G}dx = \int_0^V \mathbf{F}dV \equiv \int_{V_S}^{\infty} \mathbf{F}dV - \int_{V_D}^{\infty} \mathbf{F}dV \]

thus:

\[ I_D = I(V_S, V_G) - I(V_D, V_G) \]
The drain current is the superposition of independent and symmetrical effects of source and drain voltages.

Definitions:
- **Forward** current $I_F = I(V_S, V_G)$, independent of $V_D$
- **Reverse** current $I_R = I(V_D, V_G)$, independent of $V_S$

Then $I_D = I_F - I_R$
• Mobile charge $Q_i$ depends on surface potential $\Psi_s$, and $\Psi_s = f(V)$, thus $Q_i$ should not be a (direct) function of $x$ to be part of $F$. Therefore:

- $V_G - V_{FB}$ must be independent of position $x$ along the channel: **homogeneous channel.**
- $C_{ox}$ but may depend on $\Psi_s$ or $V$ (or $z$ for $N_b$) (e.g.: $C_{ox}(\Psi_s)$: polydepletion)

**DOMAIN OF VALIDITY (1)**

$$\mu W(-Q_i) = \frac{F(V, V_G)}{G(x, V_G)}$$

with:

$$-Q_i = C_{ox}(V_G - V_{FB} - \Psi_s) - \sqrt{2qN_b\varepsilon_{si}\Psi_s}$$

*total charge*  
*depletion charge* $Q_b$
DOMAIN OF VALIDITY (2)

- Condition: 
  \[ \mu W(-Q_i) = \frac{F(V, V_G)}{G(x, V_G)} \]

- \( W \) is independent of \( V \); thus:
  - part of \( G \), may depend on \( x \): \( \Rightarrow \) any shape of channel.

- Mobility \( \mu \) depends on vertical field thus on \( \Psi_S \), thus
  - included in \( F \), provided velocity \( v \ll v_{sat} \)
    (otherwise depends on \( I_D \) itself)

- Furthermore, the effective value of \( L \) along which \( G(x, V_G) \)
  is integrated must be independent of \( I_D, V_S \) and \( V_D \).
EFFECT OF NARROW CHANNEL

- Increased importance of side effects.
- Equivalent to parallel connection of several transistors with different characteristics.
  - if each transistor \( i \) fulfills
    \[
    I_{Di} = I_i(V_S, V_G) - I_i(V_D, V_G)
    \]
    - then the sum \( I_D \) of \( I_{Di} \) fulfills it as well.
- The property is not degraded.
The fundamental property is available

- For long and homogeneous channel
- Independently of channel shape
- Independently of \( N_b(z) \)
- Even if the channel is very narrow
- Even for large gate voltages reducing the mobility
- Even with polydepletion.
CAUSES OF DEGRADATION (1)

- Non homogeneous channel: $Q_i$ direct function of $x$.

$$Q_i = -C_{ox}(V_G - V_{FB} - \Psi_s) + \sqrt{2qN_b\varepsilon_{si}}\Psi_s$$

There may be variations with position $x$ in the channel...

- of substrate doping $N_b$, which can be
  - intentional (e.g.: LDD)
  - artifact of process (gradient or piling-up)
    (always present at very ends of channel)

- of flat-band voltage $V_{FB}$, caused by
  - variation of $N_b$
  - variation of charge in oxide

- of effective $C_{ox}$, always present at very ends of channel.
• Weak inversion characterized by $Q_i \ll Q_b$, therefore:
  • $Q_i$ has negligible effect on potential and field

• Can be expressed as $-Q_i = G_q(\Psi_s) e^{-V/UT}$
  • with $\Psi_s$ independent of $V$, thus:
    • $G_q$ can be any function of $x$ and is included in $G$, therefore:

• The property is valid even if the channel is not homogeneous.

• Mobility $\mu$ independent of $V$(small vert. field), thus part of $G$, $F$ is reduced to $F = e^{-V/UT}$: independent of $V_G$. 
CAUSES OF DEGRADATION (2)

- Channel long $\Rightarrow$ non-long $\Rightarrow$ short
  - property progressively degraded by...
  - several independent mechanisms:

a. Voltage effects:
  - channel length modulation
    - $I_F$ or $I_R$ becomes function of both $V_D$ and $V_S$
    - effect proportional to $1/L$
  - barrier lowering and 2-D effects: further degradation.

b. Current effects:
  - if $I_D$ is increased by reducing $L$, then
    $\Rightarrow$ carrier velocity increases towards saturation
    $\Rightarrow$ mobility reduced, thus function of $I_D$

c. Non-homogeneous channel (except in weak inversion):
  - importance of end-effects proportional to $1/L$. 
CONCEPT OF PSEUDO-RESISTOR

- We have shown that: 
  \[ I_D = \frac{1}{L} \int_0^L G_d x \left[ \int_{V_s}^\infty F_d V - \int_{V_D}^\infty F_d V \right] \]

- Definitions:
  - pseudo-voltage: 
    \[ V^* = -K_0 \int_{V}^\infty F(V, V_G) dV \]
  - pseudo-resistor: 
    \[ R^* = K_0 \int_0^L G(x, V_G) dx \]

(where \( K_0 \): any positive constant)

- Results in pseudo Ohm's Law:
  \[ I_D = (V_D^* - V_S^*)/R^* \]
- 14 -

LINEAR CURRENT-MODE CIRCUITS

• Implications of pseudo Ohm's law \( I_D = (V_D^* - V_S^*)/R^* \)

• Any network interconnecting transistors with same \( F(V, V_G) \) and same \( V_G \) is linear with respect to currents.

• Any circuit of linear resistors can be implemented by transistors only, provided only currents are considered.

• A resistor connected to ground \( (V=0) \) in the resistive prototype corresponds to a saturated transistor that provides a pseudo-ground \( (V^*=0) \).

• In weak inversion:
  • \( F \) indep. of \( V_G \), but \( V_G \) included in function \( G \), hence:
    • Different \( V_G \) possible for each transistor
    • Each \( R^* \) can be separately adjusted by its \( V_G \)
EXAMPLE OF APPLICATION OF PSEUDO-R

- Large-ratio current mirror
  - Use series/parallel combination of...
  - identical transistors, all in same substrate

- transistor circuit:  
  - pseudo-resistor prototype:

\[ \frac{I_2}{I_1} = 16 \]

pseudo-ground 0*

\[ \text{ground 0} \]
EXAMPLE OF APPLICATION IN WEAK INVERSION

• Calculation of harmonic mean \([7]\)

- Series combination of \(G_i\): \(G = \frac{1}{\sum 1/G_i}\) harmonic mean

- Same voltage across \(G\) and \(G_i\), thus \(I = \frac{1}{\sum 1/I_i} = \frac{I_{hm}}{N}\)
APPLICATIONS OF PSEUDO-RESISTORS

• Simplification of circuit analysis
• Linear attenuators [4] (electrical control in weak inversion)
  • R-2R network for D/A conversion [8].
• Spatial information processing:
  • $n$th oder moment computation $[9,6,10,11]$  
  • diffusion networks (isotropic or not) $[12,6]$
  • 2-D emulation of physical media $[13,6]$
• In weak inversion: exploitation of current distribution in voltage- (or current-) dependent linear networks:
  • local normalisation in vision processing $[14,6]$
  • generation of nonlinear functions $[6, 15]$
  • energy minimizers.....
CONCLUSION

• MOS property for long and homogeneous channels:

\[ I_D = I(V_S, V_G) - I(V_D, V_G) = I_F - I_R \]

• superposition of independent and symmetrical effects of S and D voltages.
• forward and reverse components.

• Property progressively degraded when channel shortened.

• Underlies the concept of pseudoresistor:
  • linear current mode circuits
  • transistor implementation of arrays of resistors.
  • simpler analysis of transistor circuits.
REFERENCES


